



#### A conclusion up front:

- Hope for *some* of today's best algorithms to make the leap
  - greater concurrency
  - less synchrony
  - built-in resilience ("algorithm-based fault tolerance" or ABFT)
- Programming models will have to be severely stretched
- Everything should be "on the table" for trades "over the threshold"

#### **Motivation**

- High performance with high productivity on "the multis":
  - Multi-scale, multi-physics problems in multidimensions
  - Using multi-models and/or multi-levels of refinement
  - Exploiting polyalgorithms in multiple precisions in multi-protocol programming styles
  - On multi-core, massively multi-processor systems
  - Requiring a multi-disciplinary approach

#### **Motivation**

- High performance with high(-est possible) productivity on "the multis":
  - Multi-scale, multi-physics problems in multidimensions
  - Using multi-models and/or multi-levels of refinement
  - Exploiting polyalgorithms in multiple precisions in multi-protocol programming styles
  - On multi-core, massively multi-processor systems
  - Requiring a multi-disciplinary approach

Can't cover all this in one talk...

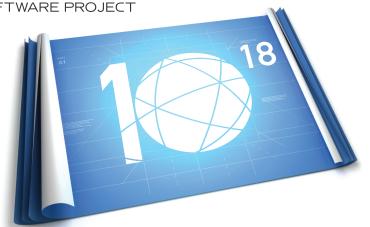
Given the architectural stresses, how can new algorithms help?

#### Purpose of the presentation

- Increase quality of "co-design" dialog between application user-developers and systems software and hardware developers
- Vendors are surprisingly willing
  - No longer one type of vendor
  - ◆ All vendors motivated by *some* mass market
    - **■** Low-power (smartphones, remote sensors, etc.)
    - High graphics throughput (gaming, entertainment, etc.)
    - High reliability (business, data centers, etc.)
  - Unfortunately, computational scientists want all three
    - ... and we are *a relatively* small market

# For background, see www.exascale.org/iesp

EXASCALE ROADMAP 1.0



Jack Dongarra
Pete Beckman
Terry Moore
Patrick Aerts
Giovanni Aloisio
Jean-Claude Andre
David Barkai
Jean-Yves Berthou
Taisuke Boku
Bertrand Braunschweig
Franck Cappello
Barbara Chapman
Xuebin Chi

Alok Choudhary Sudip Dosanjh Thom Dunning Sandro Fiore Al Geist Bill Gropp Robert Harrison Mark Hereld Michael Heroux Adolfy Hoisie Koh Hotta Yutaka Ishikawa Fred Johnson

Sanjay Kale Richard Kenway David Keyes Bill Kramer Jesus Labarta Alain Lichnewsky Thomas Lippert Bob Lucas Barney Maccabe Satoshi Matsuoka Paul Messina Peter Michielse Bernd Mohr Matthias Mueller Wolfgang Nagel Hiroshi Nakashima Michael E. Papka Dan Reed Mitsuhisa Sato Ed Seidel John Shalf David Skinner Marc Snir Thomas Sterling Rick Stevens Fred Streitz Bob Sugar Shinji Sumimoto William Tang John Taylor Rajeev Thakur Anne Trefethen Mateo Valero Aad van der Steen Jeffrey Vetter Peg Williams Roter Wisniewski Kathy Yelick The International Exascale Software Roadmap,

J. Dongarra, P. Beckman, et al., *International Journal of High Performance Computer Applications* **25**(1), 2011, ISSN 1094-3420.

SPONSORS



































### Extrapolating exponentials eventually fails

- Scientific computing at a crossroads w.r.t. extreme scale
- Proceeded steadily for decades from giga- (1988) to tera- (1998) to peta- (2008) with
  - same SPMD programming model
  - same assumptions about who is responsible for what
  - same classes of algorithms (cf. 25 yrs. of Gordon Bell Prizes)
- Exa- is qualitatively different and will be much harder
- Core numerical analysis and scientific computing will confront exascale to maintain sponsor relevance
  - not a "distraction," but an intellectual stimulus
  - potentially big gains in adapting to new hardware environment
  - the journey will be as fun as the destination

#### Relevance of exascale to users today

- Modelers are on the front line
  - without concurrent research in the form of new models and mathematics, the passage to exascale hardware will yield little new scientific fruit
- Scientists will find the computational power to do things many have wanted
  - more room for creativity in "post-forward" problems (inverse problems, control, data assimilation, uncertainty quantification)
  - scientists will participate in cross-disciplinary integration –
     "third paradigm" and "fourth paradigm"
  - remember that exascale at the lab means petascale on the desk
- We suggest some mathematical opportunities, after (quickly) reviewing the hardware challenges

### Why exa- is different

#### Which steps of FMADD take more energy?

Going across the die will require an order of magnitude more!

#### DARPA study predicts that by 2019:

- **◆ Double precision FMADD flop: 11pJ**
- cross-die per word access (1.2pJ/mm): 24pJ (= 96pJ overall)

### Today's power costs per operation

Operation	approximate energy cost
DP FMADD flop	100 pJ
DP DRAM read-to-register	4800 pJ
DP word transmit-to-neighbor	7500 pJ
DP word transmit-across-system	9000 pJ

Remember that a *pico*  $(10^{-12})$  of something done *exa*  $(10^{18})$  times per second is a *mega*  $(10^6)$ -somethings per second

- ◆ 100 pJ at 1 Eflop/s is 100 MW (for the flop/s only!)
- ♦ In the USA, 1 MW-year costs \$1M (\$0.12/KW-hr × 8760 hr/yr)
  - ♦ You "use" 1.4 KW continuously on average

#### Why exa- is different

#### Moore's Law (1965) does not end but Dennard's MOSFET scaling (1972) does

Table 1 Scaling Results for Circuit Performance

Device or Circuit Parameter	Scaling Factor
Device dimension $t_{ox}$ , $L$ , $W$	1/κ
Doping concentration $N_a$	К
Voltage V	$1/\kappa$
Current I	$1/\kappa$
Capacitance $\epsilon A/t$	$1/\kappa$
Delay time/circuit VC/I	2/1
Power dissipation/circuit VI	$1/\kappa^2$
Power density VI/A	1

Table 2
Scaling Results for Interconnection Lines

Parameter	Scaling Factor
Line resistance, $R_L = \rho L/Wt$ Normalized voltage drop $IR_L/V$	К
Line response time $R_LC$	1
ine current density I/A	К



Robert Dennard, IBM (inventor of DRAM, 1966)

Eventually processing will be limited by transmission

### What will first "general purpose" exaflop/s machines look like?

- Hardware: many potentially exciting paths beyond today's CMOS silicon-etched logic, but not commercially at scale within the decade
- Software: many ideas for general-purpose and domain-specific programming models beyond "MPI + X", but not penetrating the main CS&E workforce within the decade
  - "X" is OpenMP, CUDA, OpenACC, pthreads, etc.

### Tianhe-2 by Inspur / NUDT / Intel (June 2013)



Main system: 32K Ivy Bridge + 48K Xeon Phi chips ~55 PF/s

\*Front-end: 4K Galaxy FT-1500 chips, 59 TF/s

### Tianhe-2 vs. envisioned exascale hardware: a heterogeneous, distributed memory GigaHz KiloCore MegaNode system

	<b>Tianhe-2 (2013)</b>	Exa (2020)	Ratio to go
Number of nodes	16,000 (each 2 Ivy + 3 Phi)	1,000,000	~60
Node concurrency	24 Ivy + 171 Phi = 195 cores	1,000	~5
Node memory (GB)	88  Ivy + 8  Phi = 96	64	(1)
Node peak perf (GF/s)	422 Ivy + 3,009 Phi = 3,431	1,000	(1)
<b>Total concurrency</b>	3,120,000	1 B	~320
Total memory (PB)	1.536	64	~40
Total peak perf (PF/s)	54.9	1,000	~20
Power (MW)	17.8 (+ 24 MW cooling !)	20	(1)

#### Some exascale themes

- Clock rates cease to increase while arithmetic capacity continues to increase dramatically w/concurrency consistent with Moore's Law
- Storage capacity diverges exponentially below arithmetic capacity
- Transmission capacity (memory BW and network BW) diverges exponentially below arithmetic capacity
- Mean time between hardware interrupts shortens
- Billions of dollars of scientific software hang in the balance until better algorithms arrive to span the architectural gap

#### Implications of operating on the edge

- Draconian reduction required in power per flop and per byte will make computing and copying data less reliable
  - voltage difference between "0" and "1" will be reduced
  - circuit elements will be smaller and subject to greater physical noise per signal
  - there will be more errors that must be caught and corrected
- Power may be cycled off and on or clocks slowed and speeded based on compute schedules and based on cooling capacity
  - makes per node performance rate unreliable

#### Implications of operating on the edge

- Expanding the number of nodes (processor-memory units) beyond 10<sup>6</sup> would *not* a serious threat to algorithms that lend themselves to well-amortized precise load balancing
  - provided that the nodes are performance reliable
- A real challenge is *usefully* expanding the number of cores on a node to 10<sup>3</sup>
  - must be done while memory and memory bandwidth per node expand by (at best) ten-fold less (basically "strong" scaling)
- It is already orders of magnitude slower to to retrieve an operand from main DRAM memory than to perform an arithmetic operation will get worse by another
  - "almost all" operands must come from registers or upper cache

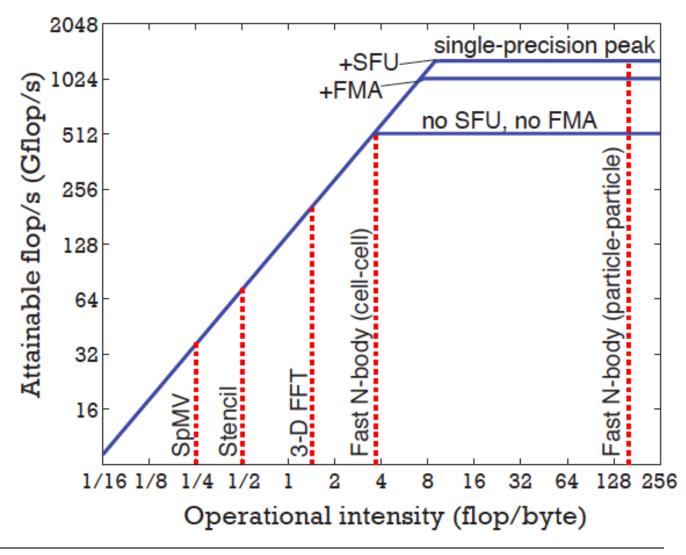
### "Missing" mathematics

- New formulations with
  - greater arithmetic intensity (flops per bytes moved into and out of registers and upper cache)
  - reduced communication
  - reduced synchronization
  - assured accuracy with (adaptively) less floatingpoint precision
  - algorithmic resilience to many types of faults
- Quantification of trades between limiting resources
- Plus all of the exciting analytical agendas that exascale is meant to exploit

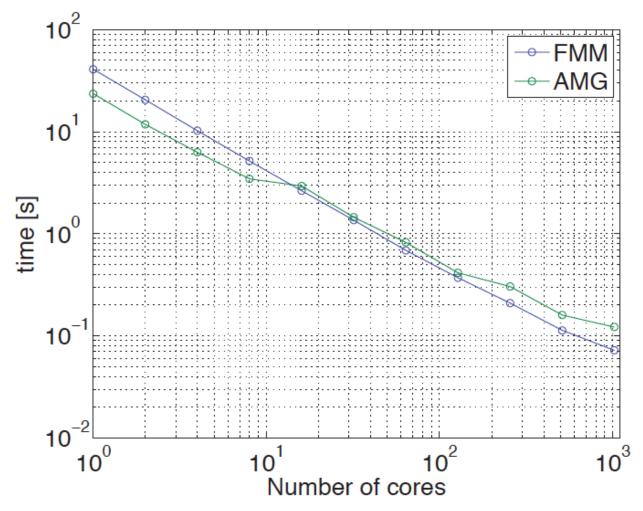
#### Arithmetic intensity illustration

Roofline model of numerical kernels on an NVIDIA C2050 GPU (Fermi). The 'SFU' label is used to indicate the use of special function units and 'FMA' indicates the use of fused multiply-add instructions.

(The order of fast multipole method expansions was set to p = 15.)



### Research in progress: FMM vs AMG preconditioning, strong scaling on Stampede\*



\* Poisson problem, Dirichlet BCs handled via BIE for FMM (cost included)

# Reduction of frequency of communication and synchronization for Ax=b

- Classical: amortize communication over many power/reduce steps
  - s-step Krylov methods: power kernels with wide halos and extra orthogonalization
  - Block Krylov methods: solve *b* several independent systems at once with improved convergence (based on  $\lambda_{max}/\lambda_b$  rather than  $\lambda_{max}/\lambda_{min}$ )
  - "tall skinny QR"  $(n \times m)$ : recursively double the row-scope of independent QRs  $\log p$  messages for p processors (vs.  $n \log p$ )
- Invade classical steps:
  - operations dynamically scheduled with DAGs
  - NUMA-aware (local) work-stealing

#### Reduction of domain of synchronization

- Nonlinear Schwarz replaces a Newton method for a global nonlinear system, F(u)=0,
  - which computes a global distributed Jacobian matrix and synchronizes globally in both the Newton step and in solving the global linear system for the Newton
- ... with a set of local problems on subsets of the global nonlinear system
  - each local problem has only local synchronization
  - all of the linear systems for local Newton updates have only local synchronization
  - there is still global synchronization in a number of steps hopefully *much fewer* than required in the original Newton method

# How are most workhorse simulations implemented at the infra-petascale today?

- Iterative methods based on data decomposition and message-passing
  - each individual processor works on a portion of the original problem and exchanges information at its boundaries with other processors that own portions with which it interacts causally, to evolve in time or to establish equilibrium
  - computation and neighbor communication are both fully parallelized and their ratio remains constant in weak scaling
- The programming model is SPMD/BSP/CSP
  - Single Program, Multiple Data
  - Bulk Synchronous Programming
  - Communicating Sequential Processes
- PETSc, Trilinos, hypre, etc.

### **Estimating scalability**

- Given complexity estimates of the leading terms of:
  - the concurrent computation (per iteration phase)
  - the concurrent communication
  - the synchronization frequency
- And a model of the architecture including:
  - internode communication (network topology and protocol reflecting horizontal memory structure)
  - on-node computation (effective performance parameters including vertical memory structure)
- One can estimate optimal concurrency and optimal execution time
  - on per-iteration basis
  - simply differentiate time estimate in terms of problem size N and processor number P with respect to P

#### 3D stencil computation weak scaling

(assume fast local network, tree-based global reductions)

Total wall-clock time per iteration (ignoring local comm.)

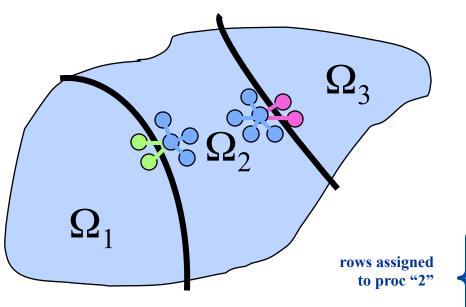
$$T(N,P) = A\frac{N}{P} + C\log P$$

 $T(N,P) = A\frac{N}{P} + C\log P$ • For optimal P,  $\frac{\partial T}{\partial P} = 0$ , or  $-A\frac{N}{P^2} + \frac{C}{P} = 0$ 

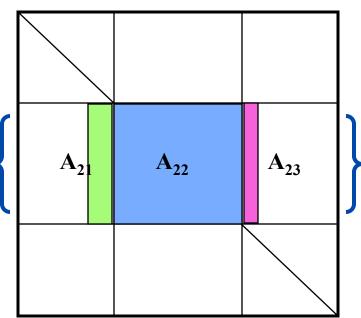
or 
$$P_{opt} = \frac{A}{C}N$$

- P can grow linearly with N, and running time increases "only" logarithmically – as good as weak scaling can be!
- **Problems:** (1) assumes perfect synchronization,
  - (2) log of a billion may be "large"

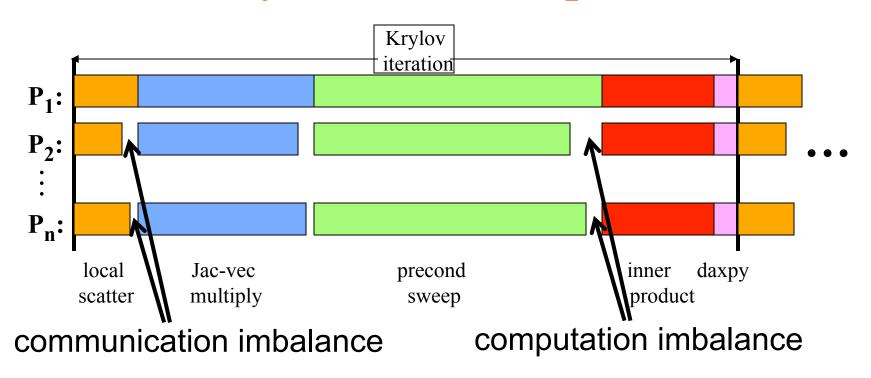
# SPMD parallelism w/ domain decomposition: an endangered species?



Partitioning of the grid induces block structure on the system matrix (Jacobian)

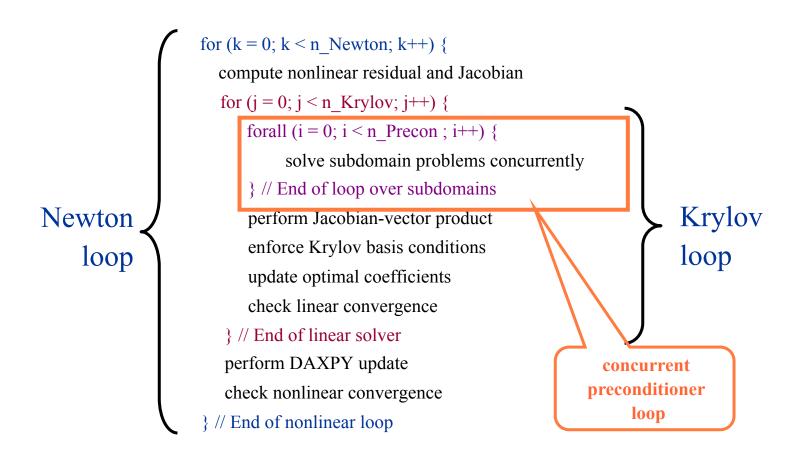


# Workhorse innards: e.g., Krylov-Schwarz, a bulk synchronous implicit solver



Idle time due to load imbalance becomes a challenge at, say, one billion cores, when *one* processor can hold up *all* of the rest at a synchronization point

### Our programming idiom is nested loops, e.g., Newton-Krylov-Schwarz



Outer loops (not shown): continuation, implicit timestepping, optimization

#### Dataflow illustration: generalized eigensolver

$$Ax = \lambda Bx$$

Operation

#### Explanation

LAPACK routine name

 $B = L \times L^T$ 

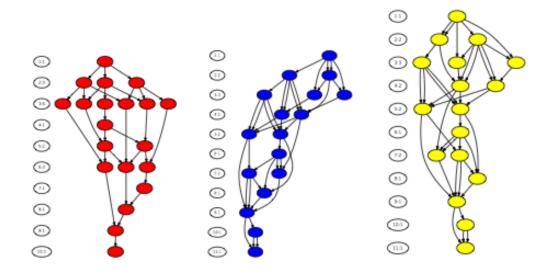
Cholesky factorization

**POTRF** 

- ${\cal C} = {\cal L}^{-1} \times {\cal A} \times {\cal L}^{-T}$  application of triangular factors SYGST or HEGST
- $Tx = \lambda x$

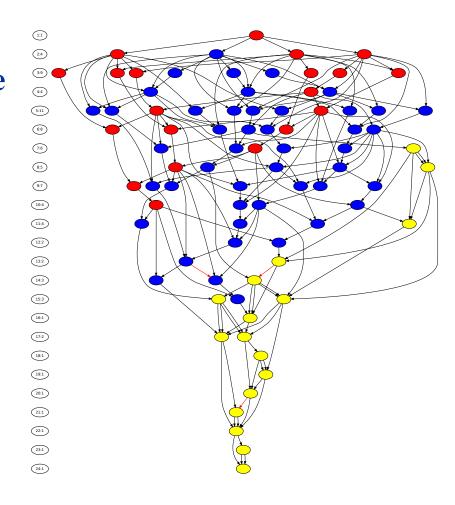
QR iteration

STERF

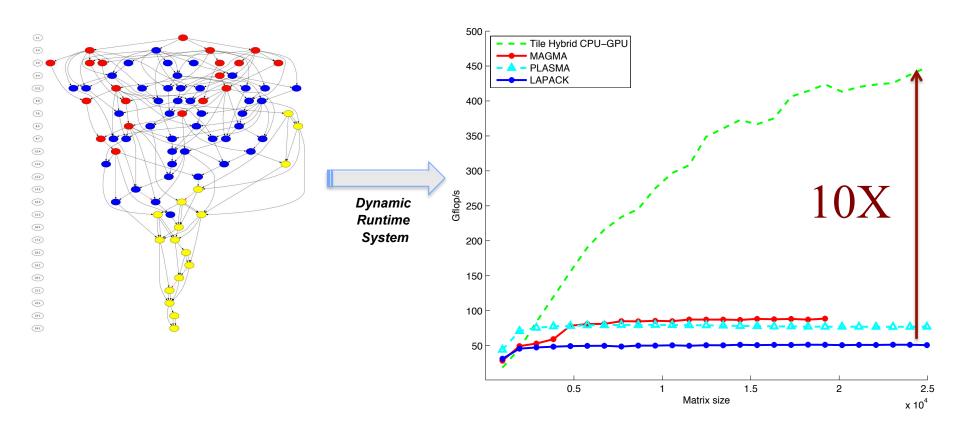


# These loops, with their artifactual orderings, can be replaced with DAGs

- Diagram shows a
   dataflow ordering of the
   steps of a 4×4
   symmetric generalized
   eigensolver
- Nodes are tasks, colorcoded by type, and edges are data dependencies
- Time is vertically downward

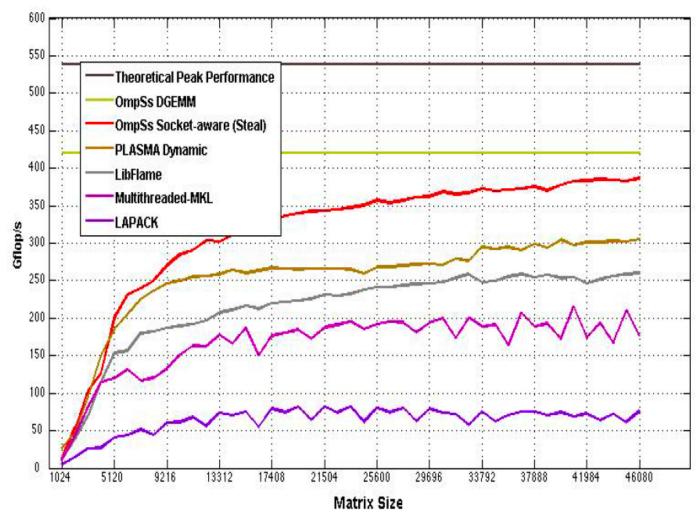


# Co-variance matrix inversion on hybrid\* CPU-GPU environment with DAG scheduling



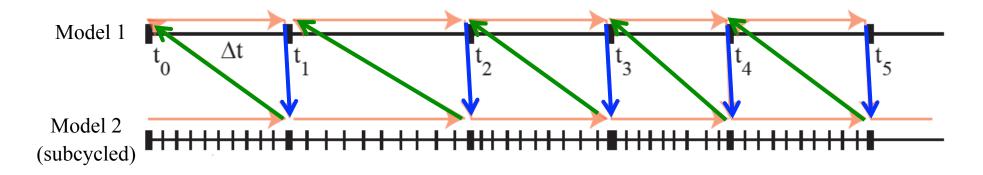
\* On 8 Intel Xeons and 2 NVIDIA Teslas using StarPU (INRIA)

### Research in progress: locality preserving workstealing on Cholesky solver gets 93% of DGEMM\*



\* On AMD "Istanbul" 8439 SE with 8 sockets, 6 cores per socket, using OmpSs

# Multiphysics w/ legacy codes: an endangered species?



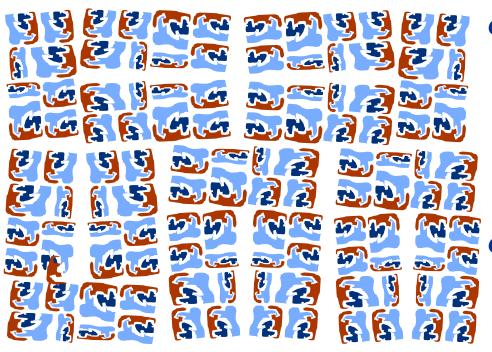
- Many multiphysics codes operate like this, where the models may occupy the same domain in the bulk (e.g., reactive transport) or communicate at interfaces (e.g., ocean-atmosphere)\*
- The data transfer cost represented by the blue and green arrows may be much higher than the computation cost of the models, even apart from first-order operator splitting error and possible instability

<sup>\*</sup>see "Multiphysics simulations: challenges and opportunities" (IJHPCA)

# Many codes have the algebraic and software structure of multiphysics

- Exascale is motivated by these:
  - uncertainty quantification, inverse problems, optimization, immersive visualization and steering
- These may carry auxiliary data structures to/from which blackbox model data is passed and they act like just another "physics" to the hardware
  - pdfs, Lagrange multipliers, etc.
- Today's separately designed blackbox algorithms for these may not live well on exascale hardware: codesign may be required due to data motion

#### Multiphysics layouts must invade blackboxes



- Each application must first be ported to extreme scale (distributed, hierarchical memory)
  - Then applications may need to be interlaced at the data structure level to minimize copying and allow work stealing at synchronization points



#### Multiphysics simulations: Challenges and opportunities

The International Journal of High Performance Computing Applications 27(1) 4-82 © The Author(s) 2012 Reprints and permissions: sagepub.co.uk/journalsPermissions.nav DOI: 10.1177/1094342012468181 hpc.sagepub.com

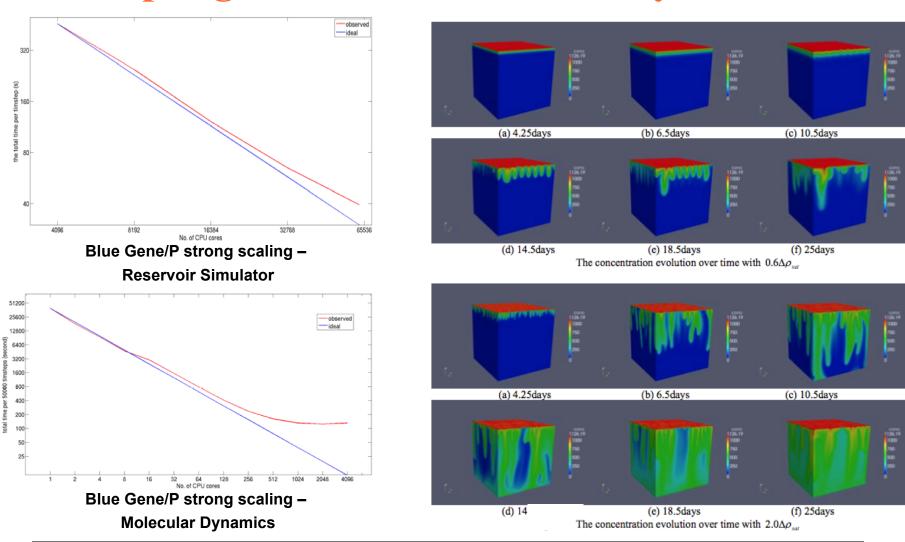
(S)SAGE

David E Keyes<sup>1,2</sup>, Lois C McInnes<sup>3</sup>, Carol Woodward<sup>4</sup>,
William Gropp<sup>5</sup>, Eric Myra<sup>6</sup>, Michael Pernice<sup>7</sup>, John Bell<sup>8</sup>,
Jed Brown<sup>3</sup>, Alain Clo<sup>1</sup>, Jeffrey Connors<sup>4</sup>, Emil Constantinescu<sup>3</sup>, Don Estep<sup>9</sup>,
Kate Evans<sup>10</sup>, Charbel Farhat<sup>11</sup>, Ammar Hakim<sup>12</sup>, Glenn Hammond<sup>13</sup>, Glen Hansen<sup>14</sup>,
Judith Hill<sup>10</sup>, Tobin Isaac<sup>15</sup>, Xiangmin Jiao<sup>16</sup>, Kirk Jordan<sup>17</sup>, Dinesh Kaushik<sup>3</sup>,
Efthimios Kaxiras<sup>18</sup>, Alice Koniges<sup>8</sup>, Kihwan Lee<sup>19</sup>, Aaron Lott<sup>4</sup>, Qiming Lu<sup>20</sup>,
John Magerlein<sup>17</sup>, Reed Maxwell<sup>21</sup>, Michael McCourt<sup>22</sup>, Miriam Mehl<sup>23</sup>,
Roger Pawlowski<sup>14</sup>, Amanda P Randles<sup>18</sup>, Daniel Reynolds<sup>24</sup>, Beatrice Rivière<sup>25</sup>,
Ulrich Rüde<sup>26</sup>, Tim Scheibe<sup>13</sup>, John Shadid<sup>14</sup>, Brendan Sheehan<sup>9</sup>, Mark Shephard<sup>27</sup>,
Andrew Siegel<sup>3</sup>, Barry Smith<sup>3</sup>, Xianzhu Tang<sup>28</sup>, Cian Wilson<sup>2</sup> and Barbara Wohlmuth<sup>23</sup>

#### Abstract

We consider multiphysics applications from algorithmic and architectural perspectives, where "algorithmic" includes both mathematical analysis and computational complexity, and "architectural" includes both software and hardware environments. Many diverse multiphysics applications can be reduced, en route to their computational simulation, to a common algebraic coupling paradigm. Mathematical analysis of multiphysics coupling in this form is not always practical for realistic applications, but model problems representative of applications discussed herein can provide insight. A variety of software frameworks for multiphysics applications have been constructed and refined within disciplinary communities and executed on leading-edge computer systems. We examine several of these, expose some commonalities among them, and attempt to extrapolate best practices to future systems. From our study, we summarize challenges and forecast opportunities.

## Multiphysics modeling of CO<sub>2</sub> sequestration by coupling PDEs and molecular dynamics



#### Bad news/good news (1)

- One may have to explicitly control data motion
  - carries the highest energy cost in the exascale computational environment
- One finally will get the privilege of controlling the vertical data motion
  - horizontal data motion under control of users under Pax MPI, already
  - but vertical replication into caches and registers was (until now with GPUs) scheduled and laid out by hardware and runtime systems, mostly invisibly to users

#### Bad news/good news (2)

- "Optimal" formulations and algorithms may lead to poorly proportioned computations for exascale hardware resource balances
  - today's "optimal" methods presume flops are expensive and memory and memory bandwidth are cheap
- Architecture may lure users into more arithmetically intensive formulations (e.g., fast multipole, lattice Boltzmann, rather than mainly PDEs)
  - tomorrow's optimal methods will (by definition) evolve to conserve what is expensive

#### Bad news/good news (3)

- Hardware nonuniformity may force abandonment of the Bulk Synchronous Programming (BSP) paradigm
  - it will be impossible for the user to control load balance sufficiently to make it work
- Hardware and algorithmic nonuniformity will be indistinguishable at the performance level
  - good solutions for the dynamically load balancing in systems space will apply to user space, freeing users

#### Bad news/good news (4)

- Fully deterministic algorithms may simply come to be regarded as too synchronization-vulnerable
  - Rather than wait for data, we may infer it, taking into account sensitivity to poor guesses, and move on
- A rich numerical analysis of algorithms that make use of statistically inferred "missing" quantities may emerge

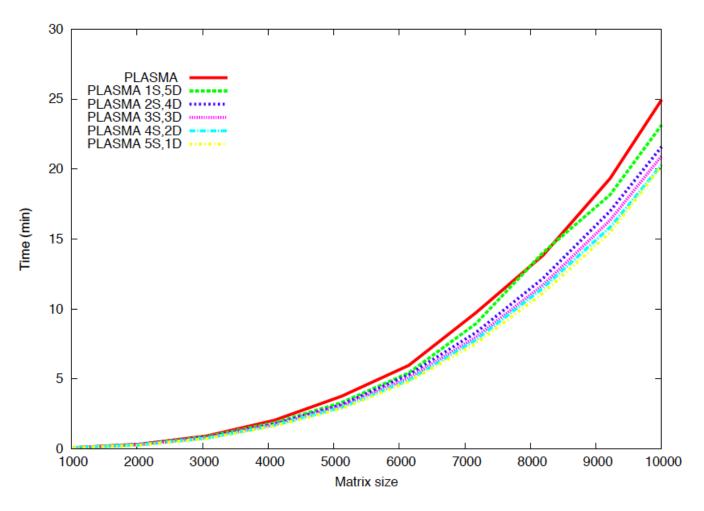
#### Bad news/good news (5)

- Fully reliable executions may simply come to be regarded as too costly/synchronization-vulnerable
- Algorithmic-based fault tolerance (ABFT)will be much cheaper than hardware and OS-mediated reliability
  - Developers will partition their data and their program units into two sets
  - A small set that must be done reliably (with today's standards for memory checking and IEEE ECC)
  - A large set that can be done fast and unreliably, knowing the errors can be either detected, or their effects rigorously bounded
- Examples in direct and iterative linear algebra
  - anticipated by Von Neumann, 1956 ("Synthesis of reliable organisms from unreliable components")

#### Bad news/good news (6)

- Default use of (uniform) high precision may come to an end, as wasteful of storage and bandwidth
  - Representation of a smooth function in a hierarchical basis requires fewer bits than storing its nodal values
  - we will have to compute and communicate "deltas" between states rather than the full state quantities, as we did when double precision was expensive (e.g., iterative correction in linear algebra)
  - a combining network node will have to remember not just the last address, but also the last values, and send just the deltas
- Equidistributing errors properly while minimizing resource use will lead to innovative error analyses in numerical analysis

## Research in progress: reducing precision in the new QDHWeig eigensolver (Higham, 2013)



\* Dual-socket 8-core (16 cores total), Intel(R) Xeon(R) CPU E5-2650

### How will PDE computations adapt?

- Programming model will still be message-passing (due to large legacy code base), adapted to multicore or hybrid processors beneath a relaxed synchronization MPI-like interface
- Load-balanced blocks, scheduled today with nested loop structures will be separated into critical and non-critical parts
- Critical parts will be scheduled with directed acyclic graphs (DAGs)
- Noncritical parts will be made available for work-stealing in economically sized chunks

# Adaptation to asynchronous programming styles

- To take full advantage of such asynchronous algorithms, we need to develop greater expressiveness in scientific programming
  - create separate threads for logically separate tasks, whose priority is a function of algorithmic state, not unlike the way a time-sharing OS works
  - join priority threads in a directed acyclic graph (DAG), a task graph showing the flow of input dependencies; fill idleness with noncritical work or steal work
- Steps in this direction
  - Asynchronous Dynamic Load Balancing (ADLB) [Lusk (Argonne),
     2009]
  - Asynchronous Execution System [Steinmacher-Burrow (IBM), 2008]

## **Evolution of Newton-Krylov-Schwarz:** breaking the synchrony stronghold

- Can write code in styles that do not require artifactual synchronization
- Critical path of a nonlinear implicit PDE solve is essentially ... lin\_solve, bound\_step, update; lin\_solve, bound\_step, update ...
- However, we often insert into this path things that could be done less synchronously, because we have limited language expressiveness
  - Jacobian and preconditioner refresh
  - convergence testing
  - algorithmic parameter adaptation
  - I/O, compression
  - visualization, data mining

#### **Sources of nonuniformity**

#### System

- Already important: manufacturing, OS jitter, TLB/cache performance variations, network contention,
- Newly important: dynamic power management, more soft errors, more hard component failures, software-mediated resiliency, etc.

#### • Algorithmic

- physics at gridcell/particle scale (e.g., table lookup, equation of state, external forcing), discretization adaptivity, solver adaptivity, precision adaptivity, etc.
- Effects of both types are similar when it comes to waiting at synchronization points
- Possible solutions for system nonuniformity will improve programmability, too

#### **Programming practice**

- Prior to possessing exascale hardware, users can prepare themselves by exploring new programming models
  - on manycore and heterogeneous nodes
- Attention to locality and reuse is valuable at all scales
  - will produce performance paybacks today and in the future
  - domains of coherence will be variable and hierarchical
- New algorithms and data structures can be explored under the assumption that flop/s are cheap and moving data is expensive

#### Path for scaling up applications

- "Weak scale" applications up to distributed memory limits
  - proportional to number of nodes
- "Strong scale" applications beyond this
  - proportional to cores per node/memory unit
- Scale the workflow, itself
  - proportional to the number of instances (ensembles)
  - integrated end-to-end simulation
- Algorithm-architecture co-design process is staged, with any of these types of scaling valuable by themselves
- Big question: does the software for co-design factor? Or is all the inefficiency at the data copies at interfaces between the components after a while?

### Required software enabling technologies

#### Model-related

- Geometric modelers
- Meshers
- Discretizers
- Partitioners
- Solvers / integrators
- Adaptivity systems
- Random no. generators
- Subgridscale physics
- Uncertainty quantification
- Dynamic load balancing
- Graphs and combinatorial algs.
- Compression

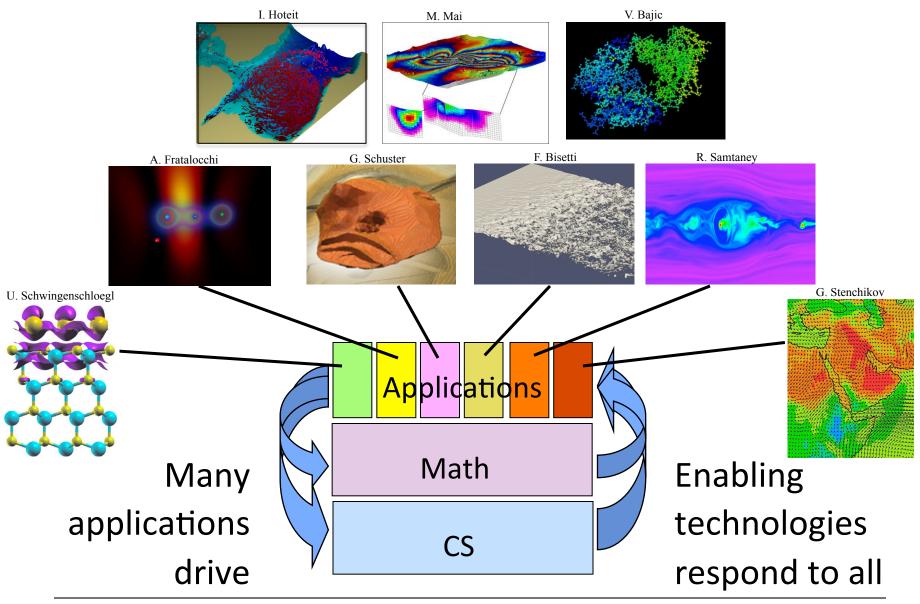
#### Development-related Production-related

- Configuration systems
- Source-to-source translators
- Compilers
- Simulators
- Messaging systems
- Debuggers
- Profilers

High-end computers comewith little of this stuff.Most has to be contributedby the user community

- Dynamic resource management
- Dynamic performance optimization
- Authenticators
- I/O systems
- Visualization systems
- Workflow controllers
- Frameworks
- Data miners
- Fault monitoring,reporting, and recovery

### "SciDAC philosophy" of software investment



#### DOE's Exascale Mathematics Working Group

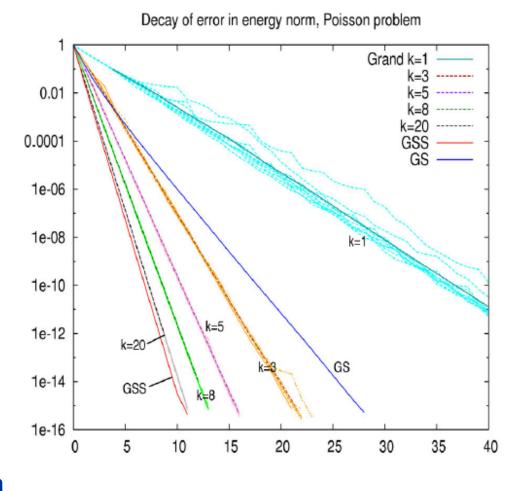
• 74 fascinating 2-page whitepapers contributed by the international community to the EMWG at

https://collab.mcs.anl.gov/display/examath/Submitted+Papers

- To be discussed this coming 20-21 August 2013 in DC
  - Randomized algorithms
  - On-the-fly data compression
  - Mining massive data sets
  - Algorithmic-based fault tolerance
  - Adaptive precision algorithms
  - Concurrency from dimensions beyond space (time, phase space, stochastic parameters)
  - etc.

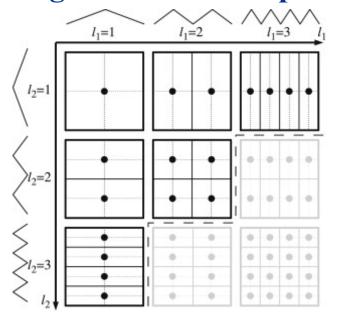
## Randomized algorithms in subspace correction methods

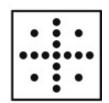
- Solve Ax=b by pointwise relaxation
- **Gauss-Seidel** (1823)
  - deterministic and preordered
- **Southwell (1935)** 
  - deterministic and dynamically ordered
- Griebel-Oswald (2011)
  - random (and dynamically ordered)
- Excellent convergence w/ fault tolerance and synchronization reduction



#### Hierarchical representations for extreme data

- Saving most simulation results to persistent storage will be impractical; instead hybrid in situ / in transit analysis
- Challenges:
  - On-the-fly compression
- Algorithmic idea: sparse grids





#### Storage complexity

$$O(n^d) \rightarrow O(n \cdot (\log n)^{d-1})$$
  
(spatial dimension d)

Representation accuracy

$$O(n^{-p}) \to O(n^{-p} \cdot (\log n)^k)$$

(order p; k depends on p, d)

### How do sparse representations work?

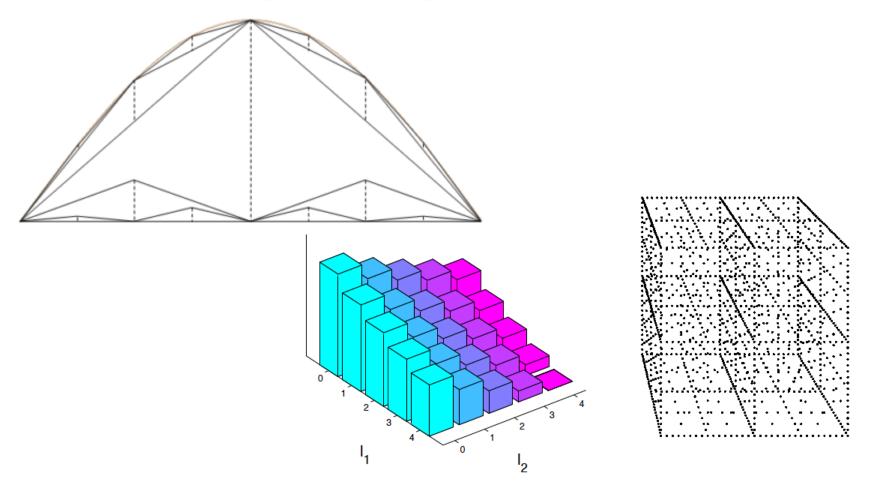


Fig. 1. Left: Norms of errors of the difference spaces  $W_1$  on a logarithmic scale. The example function is  $u(x_1, x_2) = e^{-(x_1^2 + x_2^2)}$  with  $(x_1, x_2) \in [0, 1] \times [0, 1]$ . Right: Sparse grid of refinement level l = 5 in three dimensions

# Acknowledgment: today's Peta-op/s machines



10<sup>12</sup> neurons @ 1 KHz = 1 PetaOp/s 1.4 kilograms, 20 Watts

### See 2011 special issue of Comptes Rendus

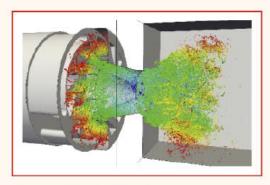
### Comptes Rendus

DE L'ACADÉMIE DES SCIENCES

fascicule 2-3

Février-mars 2011 ISSN 1631-0721

## MECANIQUE



Thematic issue / Numéro thématique High Performance Computing / Le Calcul Intensif Guest editor / Rédacteur en chef invité : Olivier Pironneau

ACADÉMIE DES SCIENCES - PARIS





Exaflop/s: The why and the how, D. E. Keyes, *Comptes Rendus de l'Academie des Sciences* **339**, 2011, 70—77.

